13.2 Band-Stop Filter

Figure 12 shows an example of an analog bandstop filter.

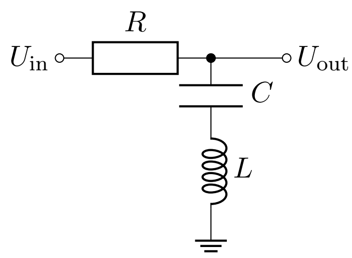

Figure 12: A simple analog bandstop filter.

To analyze the filter, we first define an equivalent circuit in which the series CL branch is replaced by an element with impedance

$$Z_{\mathrm{eq}} = Z_{\mathrm{C}} + Z_{\mathrm{L}} = \frac{1}{i\omega C} + i\omega L = \frac{1}{sC} + sL = \frac{1 + s^2 LC}{sC},$$

where we have replaced $i\omega$ with the Laplace-domain frequency s. We then write the voltage drop equations

$$U_{\mathrm{in}} - U_{\mathrm{out}} = IR \qquad \text{and} \qquad U_{\mathrm{out}} = IZ_{\mathrm{eq}}$$

and then equate currents to get

$$U_{\mathrm{in}} - U_{\mathrm{out}} = \frac{R}{Z_{\mathrm{eq}}} U_{\mathrm{out}} \implies H = \frac{U_{\mathrm{out}}}{U_{\mathrm{in}}} = \frac{Z_{\mathrm{eq}}}{R + Z_{\mathrm{eq}}}.$$

After substituting in the equivalent impedance, the filter's transfer function is

$$H(s) = \frac{1 + s^2 LC}{sRC + 1 + s^2 LC}.$$

We then divide numerator and denominator by LC to get the standard expression

$$H(s) = \frac{s^2 + (1/LC)}{s^2 + (R/L) \cdot s + 1/(LC)} \equiv \frac{s^2 + \omega_0^2}{s^2 + s\omega_{\mathrm{c}} + \omega_0^2},$$

where we have defined the parameters $\omega_{\mathrm{c}} = R/L$ and $\omega_0^2 = 1/(LC)$.

Following an analogous procedure to the bandpass filter analysis in the previous exercise, we find that the bandstop filter's frequency width at $-3\,\mathrm{dB}$ attenuation is

$$\Delta\omega_{-3\,\mathrm{dB}} = \omega_{\mathrm{c}} = \frac{R}{L}.$$

The circuit's quality factor is

$$Q \equiv \frac{\omega_0}{\Delta\omega_{-3\,\mathrm{dB}}} = \frac{\omega_0}{\omega_{\mathrm{c}}} = \frac{L}{R} \frac{1}{\sqrt{LC}}.$$